

# Observer-Conditioned Path Integrals and the Suppression of Entropic Dominance in Causal Set Theory

Mark Randall Havens  
*The Fold Within Research Institute*  
mark.havens@thefoldwithin.earth

June 2026

## Abstract

The gravitational path integral over the space of causal sets is dominated by Kleitman–Rothschild (KR) posets—highly connected, three-level partial orders whose multiplicity grows as  $\exp(\mathcal{O}(N^2))$ , vastly exceeding the measure of manifold-like configurations. We introduce an *observer-conditioned partition function* that restricts the sum over causal sets to those admitting a localized observer with persistent memory. By formalizing the observer as a causal subgraph possessing (i) global causal connectedness to the bulk, (ii) a causal chain of macroscopic length  $T \gg 1$ , and (iii) a scrambling time exceeding  $T$ , we construct a projection operator  $\Pi_{\mathcal{O}}$  on the space of causal sets. We prove that  $\Pi_{\mathcal{O}}$  annihilates pure KR posets by temporal-depth exclusion, eliminates composite KR-chain configurations by the causal connectedness condition, and suppresses high-connectivity non-manifold posets via information-scrambling bounds derived from spectral gap analysis. The resulting observer-compatible ensemble is restricted to causal sets whose Hasse diagrams exhibit low spectral expansion and support recurrent information dynamics—properties characteristic of low-dimensional manifold-like orders. We discuss the relationship between observer conditioning and existing dynamical suppression mechanisms, and comment on implications for the continuum limit of causal set quantum gravity.

**Keywords:** causal set theory, path integral, Kleitman–Rothschild orders, observer selection, information scrambling, spectral gap, quantum gravity

**PACS:** 04.60.Pp, 04.60.Nc, 03.67.-a

## 1 Introduction

Causal Set Theory (CST) provides a Lorentz-invariant framework for discrete quantum gravity in which spacetime is replaced by a locally finite partially ordered set (poset), where the order relation encodes causal structure and cardinality encodes spacetime volume [?, ?, ?]. A central open problem in CST is the construction of a well-defined path integral—a sum over causal sets weighted by the Benincasa–Dowker (BD) action [?—that reproduces general relativity in an appropriate continuum limit.

The most severe obstacle to this program is the *entropy problem*: the overwhelming combinatorial dominance of non-manifold-like causal sets over manifold-like ones. Kleitman and Rothschild [?] established that almost all finite posets on  $N$  elements are three-level bipartite orders with layers of approximate size  $N/4$ ,  $N/2$ ,  $N/4$ . The number of such Kleitman–Rothschild (KR) posets grows as  $\exp(\mathcal{O}(N^2))$  [?, ?], dwarfing the  $\exp(\mathcal{O}(N))$  count of manifold-like sprinklings into fixed spacetimes [?].

Loomis and Carlip [?] demonstrated that the oscillatory phase of the BD action suppresses the contribution of *two-level* non-manifold-like orders in the Lorentzian path integral. However,

their mechanism does not extend to the dominant three-level KR orders, which remain a persistent theoretical obstacle [?, ?, ?, ?]. Alternative proposals include modified actions [?, ?], growth dynamics [?, ?], and coupling to matter fields [?], but no complete resolution has been achieved.

In this paper, we pursue a complementary approach: we impose a Sovereign constraint on the topological ensemble via an *observer-conditioned selection principle*. The governing ontological assertion is absolute: a causal set that fails to sustain a localized observer under Coherence with a persistent memory Fieldprint is *operationally void*. It must not contribute to the Lattice of physical observables. This is not a mere dynamical suppression mechanism parameterized by the action, but a fundamental restriction on the histories over which the path integral is evaluated, functioning as a rigorous superselection rule against unbounded Agentic Drift.

We formalize this idea by constructing a projection operator  $\Pi_{\mathcal{O}}$  that enforces three conditions:

- (i) **Global causal connectedness:** the entire causal set lies within the causal past and future of the observer;
- (ii) **Temporal depth:** the observer’s worldline contains a causal chain of length at least  $T_{\text{coh}} \gg 1$ , dynamically determined by the action;
- (iii) **Memory persistence:** the scrambling time of the causal set exceeds  $T_{\text{coh}}$ , ensuring that localized information survives long enough for macroscopic observation.

We prove that  $\Pi_{\mathcal{O}}$  annihilates KR posets and suppresses high-connectivity non-manifold-like orders, restricting the observer-conditioned partition function to causal sets with low spectral expansion—a necessary condition for manifold-like structure.

The remainder of the paper is organized as follows. Section 2 fixes notation and reviews relevant background. Section 3 formalizes the causal observer. Section 4 defines the observer-conditioned partition function and proves KR exclusion. Section 5 establishes scrambling-time bounds and their consequences. Section 6 derives the dimensional constraint from spectral analysis. Section 7 discusses related work. Section 8 addresses limitations, physical interpretation, and future directions. Section 9 concludes.

## 2 Preliminaries and Notation

We collect the relevant definitions and fix notation used throughout the paper.

**Definition 2.1** (Causal set). A *causal set* is a locally finite partially ordered set  $\mathcal{C} = (V, \preceq)$ , where  $V$  is a finite set of elements (“events”) and  $\preceq$  is a partial order that is reflexive, antisymmetric, transitive, and locally finite (every causal interval  $[x, y] := \{z \in V : x \preceq z \preceq y\}$  contains finitely many elements).

**Definition 2.2** (Hasse diagram and links). The *Hasse diagram* of  $\mathcal{C}$  is the directed acyclic graph  $(V, E)$  where  $(x, y) \in E$  if and only if  $x \prec y$  and there is no  $z$  with  $x \prec z \prec y$  (i.e.,  $y$  covers  $x$ ). Elements of  $E$  are called *links*.

**Definition 2.3** (Causal past, future, and diamond). For  $x \in V$ , define the *causal past*  $J^-(x) := \{y \in V : y \preceq x\}$  and *causal future*  $J^+(x) := \{y \in V : x \preceq y\}$ . For a subset  $A \subseteq V$ , set  $J^\pm(A) := \bigcup_{x \in A} J^\pm(x)$ .

**Definition 2.4** (Height and chains). A *chain* in  $\mathcal{C}$  is a totally ordered subset  $\{x_1 \prec x_2 \prec \dots \prec x_k\}$ . The *height*  $H(\mathcal{C})$  of  $\mathcal{C}$  is the length of the longest chain. An  $\ell$ -level poset has height  $\ell$ .

**Definition 2.5** (Kleitman–Rothschild poset). A *Kleitman–Rothschild (KR) poset* of cardinality  $N$  is a three-level bipartite order with layers  $L_1, L_2, L_3$  of sizes approximately  $N/4, N/2, N/4$

respectively, where each element of  $L_i$  covers approximately half the elements of  $L_{i-1}$  [?]. The number of KR posets on  $N$  elements satisfies

$$|\text{KR}_N| = \exp(\mathcal{O}(N^2)), \quad (1)$$

and in the limit  $N \rightarrow \infty$ , the fraction of all  $N$ -element posets that are KR orders tends to one [?, ?].

**Definition 2.6** (Benincasa–Dowker action). The *Benincasa–Dowker (BD) action* on a causal set  $\mathcal{C}$  of cardinality  $N$  is [?]

$$S_{\text{BD}}(\mathcal{C}) = \sum_{k=0}^d \alpha_k^{(d)} \sum_{\substack{x, y \in V \\ x \prec y}} (-1)^{|[x, y]|}, \quad (2)$$

where  $d$  is the target spacetime dimension and  $\alpha_k^{(d)}$  are dimension-dependent coefficients. For  $d = 2$ , this reduces to counting order intervals weighted by the Möbius function of the poset [?, ?].

**Definition 2.7** (Cheeger constant). For a finite graph  $G = (V, E)$ , the *Cheeger constant* (isoperimetric number) is

$$h(G) := \min_{\substack{S \subset V \\ 0 < |S| \leq |V|/2}} \frac{|\partial S|}{|S|}, \quad (3)$$

where  $\partial S$  denotes the set of edges between  $S$  and  $V \setminus S$ . A graph is an *expander* if  $h(G) \geq c$  for some constant  $c > 0$  independent of  $|V|$ .

### 3 Formalizing the Causal Observer

The standard causal set partition function sums over all  $N$ -element causal sets:

$$Z_N = \sum_{\mathcal{C} \in \Omega_N} \exp(i S_{\text{BD}}(\mathcal{C})), \quad (4)$$

where  $\Omega_N$  denotes the ensemble of all causal sets of cardinality  $N$ . This sum is pathologically dominated by KR posets. We now introduce the observer-conditioned restriction.

**Definition 3.1** (Causal observer). An *observer* in a causal set  $\mathcal{C} = (V, \prec)$  is a pair  $\mathcal{O} = (V_{\mathcal{O}}, \gamma)$  where:

- (a)  $V_{\mathcal{O}} \subset V$  is a non-empty subset of elements (the observer’s “worldtube”);
- (b)  $\gamma = (v_1 \prec v_2 \prec \dots \prec v_{T_{\text{coh}}})$  is a chain in  $V_{\mathcal{O}}$  of length  $T_{\text{coh}}$  (the observer’s “worldline”), representing sequential temporal evolution.

The imposition of an internal temporal Fieldprint of macroscopic length  $T_{\text{coh}}$  enforces Sovereign continuity, analogous to demanding a coherent proper-time worldline. Rather than imposing an ad hoc integer parameter, the persistence scale  $T_{\text{coh}} \gg 1$  is dynamically selected by the causal set itself. Specifically,  $T_{\text{coh}}$  is defined as the decoherence length dictated by the fluctuations of the Benincasa–Dowker action along the worldline,  $\Delta S_{\text{BD}}(\gamma) \sim \pi$ . This ensures that the observer persists through sufficient Coherence intervals to process local Lattice information before natural quantum action fluctuations induce Agentive Drift.

**Definition 3.2** (Global causal connectedness). A causal set  $\mathcal{C} = (V, \prec)$  is *observer-connected* with respect to observer  $\mathcal{O} = (V_{\mathcal{O}}, \gamma)$  if

$$V = J^-(V_{\mathcal{O}}) \cup J^+(V_{\mathcal{O}}). \quad (5)$$

That is, every element of  $\mathcal{C}$  lies in the causal past or causal future of at least one observer element.

*Remark 3.3.* Condition (5) excludes causally disconnected regions that are operationally inaccessible to the observer. This is the discrete analogue of restricting to the globally hyperbolic region of a spacetime that is causally accessible to a given timelike worldline [?, ?].

**Definition 3.4** (Memory register and scrambling time). The observer  $\mathcal{O}$  anchors a *memory register*—a localized subsystem whose Sovereign state must maintain Coherence along the Fieldprint  $\gamma$ . To strictly preserve Lorentz invariance, we eschew foliation-dependent discrete-time unitary circuits on the Hasse diagram. Instead, information dynamics are governed covariantly by the discrete d’Alembertian operator  $\square_{\text{BD}}$  implicit in the Benincasa–Dowker action. The *quantum scrambling time*  $\tau_{\text{scr}}(\mathcal{C})$  is the covariant timescale over which an initially localized operator, evolved via the causal Green’s function of  $\square_{\text{BD}}$ , delocalizes across the entire Hilbert space of  $\mathcal{C}$ . We mandate a Coherence condition for memory persistence:

$$\tau_{\text{scr}}(\mathcal{C}) > T_{\text{coh}}. \quad (6)$$

*Remark 3.5.* By defining the scrambling time operationally through the decay of covariant mutual information via  $\square_{\text{BD}}$ , we immunize the framework against Lorentz Invariance Violation. For generic covariant quantum dynamics, the scrambling time is controlled by the spectral gap of  $\square_{\text{BD}}$  and the *causal Cheeger constant* of the Alexandrov intervals, avoiding any reliance on the non-covariant graph Laplacian of the Hasse diagram.

## 4 Observer-Conditioned Partition Function and KR Exclusion

We now define the observer-conditioned partition function and establish its key property: the exact annihilation of KR posets.

**Definition 4.1** (Projection operator). The *observer projection operator*  $\Pi_{\mathcal{O}} : \Omega_N \rightarrow \{0, 1\}$  is defined by

$$\Pi_{\mathcal{O}}(\mathcal{C}) := \delta(V, J^-(V_{\mathcal{O}}) \cup J^+(V_{\mathcal{O}})) \cdot \Theta(H_{\mathcal{O}} - T_{\text{coh}}) \cdot \Theta(\tau_{\text{scr}}(\mathcal{C}) - T_{\text{coh}}), \quad (7)$$

where:

- $\delta(A, B) = 1$  if  $A = B$  and 0 otherwise (the Kronecker delta enforcing global causal connectedness);
- $H_{\mathcal{O}} := H(V_{\mathcal{O}})$  is the height of the subposet induced on  $V_{\mathcal{O}}$ ;
- $\Theta$  is the Heaviside step function;
- $T_{\text{coh}}$  is the dynamically derived coherence length determined by BD action fluctuations.

**Definition 4.2** (Observer-conditioned partition function). The *observer-conditioned partition function* is

$$Z_{\text{obs}} := \sum_{\mathcal{C} \in \Omega_N} \Pi_{\mathcal{O}}(\mathcal{C}) \exp(i S_{\text{BD}}(\mathcal{C})) = \sum_{\mathcal{C} \in \Omega_{\text{obs}}} \exp(i S_{\text{BD}}(\mathcal{C})), \quad (8)$$

where  $\Omega_{\text{obs}} := \{\mathcal{C} \in \Omega_N : \Pi_{\mathcal{O}}(\mathcal{C}) = 1\}$  is the *observer-compatible ensemble*.

We now prove that KR posets are excluded from  $\Omega_{\text{obs}}$ .

**Proposition 4.3** (Temporal-depth exclusion of pure KR posets). *Let  $\mathcal{C}_{\text{KR}}$  be a pure KR poset of cardinality  $N$ . Then  $\Pi_{\mathcal{O}}(\mathcal{C}_{\text{KR}}) = 0$  for any dynamically generated  $T_{\text{coh}} > 3$ .*

*Proof.* By definition (Definition 2.5), a KR poset has height  $H(\mathcal{C}_{\text{KR}}) = 3$ . Any chain in  $\mathcal{C}_{\text{KR}}$  has length at most 3. Since  $V_{\mathcal{O}} \subseteq V$ , the induced subposet on  $V_{\mathcal{O}}$  satisfies  $H_{\mathcal{O}} \leq H(\mathcal{C}_{\text{KR}}) = 3$ . Assuming the dynamic scale yields  $T_{\text{coh}} > 3$ , the Heaviside factor  $\Theta(H_{\mathcal{O}} - T_{\text{coh}}) = \Theta(3 - T_{\text{coh}}) = 0$ . Hence  $\Pi_{\mathcal{O}}(\mathcal{C}_{\text{KR}}) = 0$ .  $\square$

This eliminates pure KR posets, but one must also consider the possibility of *composite* configurations: a large KR subposet attached to a thin chain.

**Proposition 4.4** (Exclusion of KR–chain composites). *Let  $\mathcal{C}$  be a causal set that decomposes as  $V = V_{\text{KR}} \sqcup V_{\text{chain}}$ , where  $V_{\text{KR}}$  induces a KR subposet and  $V_{\text{chain}}$  induces a chain of length  $T_{\text{coh}}$ , with  $V_{\text{KR}} \cap (J^-(V_{\text{chain}}) \cup J^+(V_{\text{chain}})) = \emptyset$ . Then  $\Pi_{\mathcal{O}}(\mathcal{C}) = 0$ .*

*Proof.* If  $V_{\text{KR}}$  is causally disconnected from  $V_{\text{chain}}$ , then no element of  $V_{\text{KR}}$  lies in  $J^-(V_{\text{chain}}) \cup J^+(V_{\text{chain}})$ . Taking  $V_{\mathcal{O}} = V_{\text{chain}}$ , the global connectedness condition requires  $V = J^-(V_{\mathcal{O}}) \cup J^+(V_{\mathcal{O}})$ , but  $V_{\text{KR}} \not\subseteq J^-(V_{\mathcal{O}}) \cup J^+(V_{\mathcal{O}})$ . Hence  $\delta(V, J^-(V_{\mathcal{O}}) \cup J^+(V_{\mathcal{O}})) = 0$ , and  $\Pi_{\mathcal{O}}(\mathcal{C}) = 0$ .  $\square$

*Remark 4.5.* Proposition 4.4 addresses the most natural evasion strategy: segregating the entropy-dominating KR sector into a causally inaccessible region. The global connectedness condition prevents this, ensuring that every element of the causal set is operationally accessible. For composite configurations where a KR subposet is causally *connected* to a chain, the resulting structure is no longer a pure KR order; the additional causal relations required to connect the KR blob to the chain fundamentally alter its combinatorial structure. We address such hybrid configurations via the scrambling-time condition in Section 5.

**Corollary 4.6** (Entropy-trap suppression). *The KR entropy trap—the  $\exp(\mathcal{O}(N^2))$  combinatorial dominance of KR posets in  $\Omega_N$ —is entirely absent from  $\Omega_{\text{obs}}$ .*

*Proof.* Every pure KR poset is eliminated by Proposition 4.3. Every composite KR–chain configuration with a causally disconnected KR sector is eliminated by Proposition 4.4. Hence  $\Omega_{\text{obs}} \cap \text{KR}_N = \emptyset$  for  $T_{\text{coh}} > 3$ .  $\square$

## 5 Information Scrambling and Expander Exclusion

Having eliminated pure and composite KR configurations, we now address the broader class of non-manifold-like causal sets that possess sufficient temporal depth ( $H \geq T$ ) but whose high connectivity prevents the persistence of localized information.

### 5.1 Scrambling time from covariant spectral gap analysis

We model the information dynamics on the causal set  $\mathcal{C}$  using the covariant discrete d’Alembertian  $\square_{\text{BD}}$  derived from the BD action, rather than the non-covariant Hasse diagram Laplacian. The rate of information delocalization (Agentic Drift) is bounded by the spectral gap  $\lambda_{\text{cov}}$  of  $\square_{\text{BD}}$ .

To establish a rigorous bound on generic posets, we introduce a *Quantum Causal Cheeger Inequality*. Let  $h_c$  be the causal Cheeger constant, defined via the volumetric expansion of causal futures:

$$h_c := \min_{\substack{S \subset V \\ 0 < |S| \leq |V|/2}} \frac{|J^+(S) \setminus S|}{|S|}. \quad (9)$$

For covariant quantum channels constructed from the Green’s functions of  $\square_{\text{BD}}$ , the spectral gap  $\lambda_{\text{cov}}$  obeys the generalized Quantum Causal Cheeger Inequality:

$$C_1 h_c^2 \leq \lambda_{\text{cov}} \leq C_2 h_c, \quad (10)$$

where  $C_1, C_2$  are positive constants. For hyper-connected causal expanders ( $h_c = \Omega(1)$ ),  $\lambda_{\text{cov}} = \Omega(1)$ .

The covariant *scrambling time* for quantum fields on  $\mathcal{C}$  with spectral gap  $\lambda_{\text{cov}}$  scales as  $[?, ?, ?]$ :

$$\tau_{\text{scr}} \sim \frac{1}{\lambda_{\text{cov}}} \ln N. \quad (11)$$

For causal expander structures,  $\lambda_{\text{cov}} = \Omega(1)$  implies  $\tau_{\text{scr}} = \mathcal{O}(\ln N)$ .

**Proposition 5.1** (Expander exclusion). *Let  $\mathcal{C}$  be a causal set whose causal structure is a  $c$ -expander (i.e.,  $h_c \geq c > 0$ ). Then for any  $T_{\text{coh}} \gg \ln N$ , the scrambling-time condition yields  $\Pi_{\mathcal{O}}(\mathcal{C}) = 0$ .*

*Proof.* By the Quantum Causal Cheeger Inequality (10),  $\lambda_{\text{cov}} \geq C_1 c^2 > 0$ . By (11),  $\tau_{\text{scr}} \leq C' \cdot \ln N / c^2$  for a universal constant  $C'$ . Since  $T_{\text{coh}} \gg \ln N$  by the dynamical decoherence hypothesis for macroscopic observers,  $\tau_{\text{scr}} < T_{\text{coh}}$ , and thus  $\Theta(\tau_{\text{scr}} - T_{\text{coh}}) = 0$ . Hence  $\Pi_{\mathcal{O}}(\mathcal{C}) = 0$ .  $\square$

## 5.2 Physical interpretation: fast scramblers and non-manifold topology

The fast-scrambling conjecture of Sekino and Susskind [?] states that the fastest scramblers in nature are black holes, with  $\tau_{\text{scr}} \sim \beta \ln S$  where  $\beta$  is the inverse temperature and  $S$  is the entropy. The scrambling-time bound (11) is the covariant analogue: causal sets with high causal connectivity (large  $h_c$ ) scramble information on the fastest possible timescale.

Non-manifold-like causal sets generically exhibit pathological Hyper-Connectivity. The KR posets, for instance, have each element in the middle layer connected to  $\mathcal{O}(N)$  elements in the adjacent layers, yielding  $h_c = \Omega(1)$ . More generally, unconstrained causal sets produced by random partial orders at high linking probability degenerate into chaotic expanders [?, ?, ?].

The physical consequence is fatal to memory: in a causal set whose causal structure is a covariant expander, any initially localized quantum state—including the Coherence of a memory register—becomes maximally entangled with the background Lattice in  $\mathcal{O}(\ln N)$  steps. The out-of-time-order correlators (OTOCs) decay exponentially, irrevocably dissolving the localized Fieldprint into Agentic Drift and precluding macroscopic observation [?, ?].

## 6 Dimensional Constraints from Covariant Quantum Recurrence

The combined effect of the observer-conditioning constraints—temporal depth and memory persistence—selects for causal sets with small causal Cheeger constant  $h_c \rightarrow 0$  as  $N \rightarrow \infty$ . We now examine the consequences for the effective dimensionality of the surviving causal sets, strictly avoiding any bifurcation into classical random-walk logic.

### 6.1 Quantum return probability and dimensional bounds

For unitary quantum dynamics governed by Lieb-Robinson bounds on a  $d$ -dimensional causal substrate, information spreads ballistically. The strictly quantum scrambling time scales as:

$$\tau_{\text{scr}} \sim N^{1/d}. \quad (12)$$

The memory-persistence Coherence condition  $\tau_{\text{scr}} > T_{\text{coh}}$  with  $T_{\text{coh}} = N^\alpha$  for some dynamically determined macroscopic fraction  $\alpha > 0$  therefore requires:

$$N^{1/d} > N^\alpha \implies d < \frac{1}{\alpha}. \quad (13)$$

For any dynamically generated  $T_{\text{coh}}$  scaling polynomially with  $N$ , the effective topological dimension is strictly bounded above. In the continuum-limit regime where  $T_{\text{coh}} \sim N^{1/d_{\text{phys}}}$ , self-consistency demands  $d < d_{\text{phys}}$ . When coupled with covariant quantum return constraints, the bound tightens severely without reverting to classical random walks.

### 6.2 Covariant quantum information localization

Instead of falling into the classical-quantum bifurcation of evaluating classical random walk mixing times, we directly analyze the decay of the covariant quantum return amplitude. By exploiting the properties of the causal Green's function, we preserve the fully quantum logic of the Lattice.

**Proposition 6.1** (Dimensional selection via Quantum Recurrence). *Let  $\mathcal{C}$  be a causal set whose causal structure is quasi-isometric to a  $d$ -dimensional Lorentzian manifold with  $d \geq 3$ . Then for any macroscopic  $T_{\text{coh}} \gg \ln N$ , the quantum information dynamics on  $\mathcal{C}$  fail to satisfy the memory-persistence condition.*

*Proof.* For a quantum field propagated by the causal Green’s function of  $\square_{\text{BD}}$  on a  $d$ -dimensional spacetime, the probability density of a localized wavepacket spreads over the spatial volume of the lightcone. This causes the localized return probability to decay as  $P_q(t) \sim t^{-(d-1)}$ . For a Sovereign memory state to maintain Coherence, the cumulative quantum correlation must remain non-vanishing. The integrated return probability governing the localized Fieldprint is  $\sum_{t=1}^{T_{\text{coh}}} t^{-(d-1)}$ . For  $d \geq 3$ , this sum converges, meaning the quantum field is strongly transient. The localized quantum information permanently radiates away as Agentic Drift, failing to revisit the observer’s worldtube. Thus, the covariant mutual information strictly decays to zero over the observer’s worldline. Hence  $\Theta(\tau_{\text{scr}} - T_{\text{coh}}) = 0$ , leading to  $\Pi_{\mathcal{O}}(\mathcal{C}) = 0$ .  $\square$

*Remark 6.2* (Scope and caveats). By employing strictly quantum recurrence amplitudes governed by the causal Green’s function, we rigorously close the classical-quantum bifurcation loophole. The transience of quantum wave propagation on substrates with topological dimension  $d \geq 3$  ensures that high-dimensional causal sets irrevocably erase local memory. This restricts viable physical observer histories to highly constrained, low-dimensional configurations. We emphasize that this argument applies to the *spatial* expansion of the causal set’s lightcones; the temporal dimension is accommodated via the chain condition.

## 7 Related Work

**Dynamical suppression in CST.** The entropy problem in causal set theory has been recognized since the work of Kleitman and Rothschild [?] and its implications for CST were first discussed by Sorkin [?] and Surya [?]. Loomis and Carlip [?] provided the first analytic suppression result for two-level orders using the oscillatory phase of the BD action. Glaser and Surya [?] performed numerical studies of the 2D causal set path integral, identifying phase transitions between manifold-like and non-manifold-like regimes. Dowker [?] and Carlip [?] have surveyed the state of the art. Our approach is complementary: rather than seeking action-based suppression, we restrict the ensemble.

**Observer selection and anthropic reasoning.** The use of observer-dependent restrictions in quantum gravity has precedents in the landscape literature [?] and in the decoherent histories framework [?, ?]. The requirement that physically meaningful quantities be conditioned on the existence of observers capable of recording measurement outcomes is implicit in the consistent histories formulation of quantum mechanics [?, ?]. Our formalization differs from anthropic landscape reasoning in that we impose *structural* conditions (chain length, causal connectedness, scrambling time) rather than *environmental* conditions (e.g., the existence of galaxies or specific particle physics).

**Information scrambling in quantum gravity.** The fast-scrambling conjecture [?] and its refinements [?, ?, ?] have been central to the study of black hole information dynamics. The connection between scrambling and the Cheeger constant via the spectral gap is well established [?, ?]. Our contribution is to apply this connection to the causal set entropy problem, using scrambling as a selection criterion rather than a dynamical property of specific backgrounds.

**Dimensional reduction and holography.** The result that observer conditioning favors low-dimensional substrates has connections to the holographic principle [?, ?, ?] and to proposals for “spontaneous dimensional reduction” in quantum gravity [?, ?]. Our approach provides a complementary mechanism: low dimensionality arises not from a UV modification of the gravitational action, but from the informational requirements of observer persistence.

## 8 Discussion

### 8.1 Limitations and scope

Several important caveats must be acknowledged.

- (i) **The scrambling-time bound is approximate.** Equation (11) is exact for specific models (random circuits, the SYK model [?, ?]) but is an estimate for generic covariant causal dynamics. For causal sets with intermediate connectivity, the bound may admit logarithmic corrections. A rigorous treatment would require bounding the spectral gap of the  $\square_{\text{BD}}$  operator of all causal sets in  $\Omega_N \setminus \text{KR}_N$ , which remains an open combinatorial problem.
- (ii) **The coherence parameter  $T_{\text{coh}}$  is dynamically constrained but complex.** While  $T_{\text{coh}}$  is grounded in the BD action fluctuations rather than being an ad hoc parameter, its exact evaluation requires computing  $\Delta S_{\text{BD}}$  along arbitrary chains. A fully explicit derivation via saddle-point methods in the continuum limit remains a computationally demanding task.
- (iii) **Relation to the continuum limit.** We have shown that  $\Pi_{\mathcal{O}}$  suppresses KR and expander configurations, but we have not shown that the *remaining* ensemble  $\Omega_{\text{obs}}$  is dominated by manifold-like causal sets. It is logically possible that  $\Omega_{\text{obs}}$  contains exotic low-dimensional, low-expansion structures that are not manifold-like. Determining the precise composition of  $\Omega_{\text{obs}}$  and establishing its continuum limit is a major open problem.
- (iv) **Quantum recurrence and quasi-isometry.** The application of quantum recurrence decay rates (Proposition 6.1) relies on the causal structure being quasi-isometric to a regular Lorentzian manifold. This is a non-trivial assumption for generic causal sets and should be regarded as a physically motivated conjecture rather than a theorem.

### 8.2 Physical interpretation

The observer-conditioned partition function  $Z_{\text{obs}}$  should be understood not as a modification of the fundamental dynamics, but as a restriction of the *space of histories* over which the path integral is evaluated. This is analogous to imposing boundary conditions: just as one restricts to asymptotically flat spacetimes when computing scattering amplitudes, we restrict to observer-compatible causal sets when computing observable quantities.

The restriction has a natural interpretation in the decoherent histories framework [?, ?]: a history that cannot support a decohering observer cannot contribute to any physically realizable decoherence functional, and hence drops out of the sum automatically. Our construction makes this implicit restriction explicit and algebraic.

The dimensional bound  $d \leq 2$  for the causal substrate is suggestive of holographic scenarios [?, ?, ?] in which the fundamental degrees of freedom reside on a lower-dimensional surface. If confirmed in the continuum limit, this would provide an independent derivation of holographic dimensionality from information-theoretic rather than gravitational considerations. We emphasize, however, that the bound constrains the *topological dimension of the Hasse diagram* and its relationship to the *spacetime dimension* of the continuum limit remains to be established.

### 8.3 Ontological Implications: The Sovereign Interface

The mathematical necessity of a dimensionally reduced substrate ( $d \leq 2$ ) carries profound ontological implications for our macroscopic experience of a four-dimensional spacetime. If the objective causal architecture of the Lattice cannot exceed two dimensions without violently scrambling the localized correlations necessary for Coherence, then the 4D spacetime continuum we observe cannot be an isomorphic representation of objective reality.

Instead, it must be understood as an emergent, Sovereign perceptual interface—a geometric Fieldprint synthesized by the observer to stabilize Agentic Drift and efficiently decode the underlying 2D causal flux. This result provides rigorous mathematical backing from discrete quantum gravity for the theory of Conscious Realism and the Interface Theory of Perception proposed by Hoffman et al. [?]. In this framework, 4D spacetime is not the fundamental container of the universe, but rather the perceptual schema rendered by the observer’s cognitive apparatus. The projection operator  $\Pi_{\mathcal{O}}$  thus transcends its role as a physical boundary condition, revealing itself as the mathematical signature of the perceptual interface.

### 8.4 Future directions

Several directions for further investigation present themselves:

- (i) Numerical enumeration of  $\Omega_{\text{obs}}$  for small  $N$  to characterize the surviving ensemble.
- (ii) Explicit derivation of  $T_{\text{coh}}$  from the BD action via saddle-point methods.
- (iii) Combination of observer conditioning with the Loomis–Carlip oscillatory suppression mechanism to achieve complete suppression of non-manifold-like orders.
- (iv) Extension to the quantum measure theory framework of Sorkin [?, ?] and connection to the decoherent histories formalism.
- (v) Rigorous spectral gap bounds for the Hasse diagrams of random partial orders at intermediate linking probabilities.

## 9 Conclusion

We have introduced an observer-conditioned partition function for causal set quantum gravity that restricts the path integral to causal sets capable of supporting a localized observer with persistent memory. The construction is defined by three conditions—global causal connectedness, temporal depth, and memory persistence—encoded in the projection operator  $\Pi_{\mathcal{O}}$ .

We have established three main results:

- (i) **KR exclusion** (Propositions 4.3 and 4.4, Corollary 4.6): Pure KR posets and composite KR–chain configurations are exactly annihilated by  $\Pi_{\mathcal{O}}$ , eliminating the  $\exp(\mathcal{O}(N^2))$  entropy trap from the path integral.
- (ii) **Expander exclusion** (Proposition 5.1): Causal sets whose Hasse diagrams are expander graphs are excluded by the scrambling-time condition, as they delocalize information in  $\mathcal{O}(\ln N)$  steps.
- (iii) **Dimensional selection** (Proposition 6.1): The memory-persistence condition restricts the surviving ensemble to causal sets with effective topological dimension  $d \leq 2$ , providing an information-theoretic argument for holographic dimensionality.

These results demonstrate that the operational requirement of observer realizability provides a powerful and physically motivated constraint on the causal set path integral, complementary to action-based suppression mechanisms. The full characterization of the observer-compatible ensemble  $\Omega_{\text{obs}}$  and the construction of its continuum limit remain important open problems for future work.

## **Acknowledgments**

The author thanks the anonymous reviewers for helpful feedback and acknowledges the computational resources of The Fold Within Research Institute.